## Example:

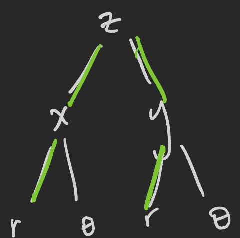
$$z = f(x,y) = \sqrt{1+xy}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Compute $\dfrac{\partial z}{\partial r}$ using the chain rule.

## Solution:



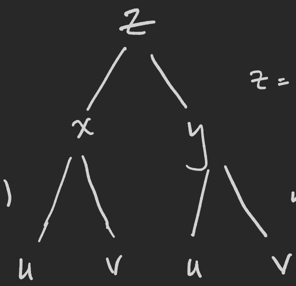

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}$$

$$= \frac{1}{2}(1+xy)^{-\frac{1}{2}}\, y\cos\theta$$

$$+ \frac{1}{2}(1+xy)^{-\frac{1}{2}}\, x\sin\theta$$

If wanted/needed, can express entirely in terms of $r, \theta$.

## Example:




$z = f(x,y)$

$x = g(u,v)$ $\quad$ $y = h(u,v)$

Compute $\dfrac{\partial z}{\partial u \partial v}$ in terms of $f, g, h$ and their partial derivatives.

Rmk $\dfrac{\partial z}{\partial u\partial v} = \dfrac{\partial}{\partial u}\left(\dfrac{\partial z}{\partial v}\right)$. (By Clairaut the order doesn't really matter.)

$$\frac{\partial z}{\partial v} = f_x(x,y)\, g_v(u,v) + f_y(x,y)\, h_v(u,v)$$

$$\frac{\partial z}{\partial u \partial v} = \frac{\partial}{\partial u}\left(f_x(x,y)\, g_v(u,v)\right)$$

$$+ \frac{\partial}{\partial u}\left(f_y(x,y)\, h_v(u,v)\right)$$

$$= \boxed{\left(\frac{\partial}{\partial u} \boxed{f_x(x,y)}\right)} g_v(u,v) + f_x(x,y)\frac{\partial}{\partial u} g_v(u,v)$$

$$+ \boxed{\left(\frac{\partial}{\partial u} f_y(x,y)\right)} h_v(u,v) + f_y(x,y)\frac{\partial}{\partial u} h_v(u,v)$$

So need chain rule again

$$\begin{tikzcd}[column sep=small]
 & & f_x(x,y) \arrow[dl, dash] \arrow[dr, dash] & & \\
 & x \arrow[dl, dash] \arrow[d, dash] & & y \arrow[d, dash] \arrow[dr, dash] & \\
u & v & & u & v
\end{tikzcd}$$

$$= \boxed{\left(f_{xx}(x,y)\, g_u(u,v) + f_{xy}(x,y)\, h_u(u,v)\right)}$$

$$\cdot\, g_v(u,v) + f_x(x,y)\, g_{vu}(u,v)$$

$$+ \boxed{\phantom{xxxxxxxxxx}}\, h_v(u,v) + f_y(x,y)\, h_{vu}(u,v)$$

Q: Difference between $d$ and $\partial$?

Ex) $z = x^2 y^2$

(Suppose also that $y = x^2$.)

$$\frac{dz}{dx} = \frac{d}{dx}\left(x^6\right) = 6x^5 \qquad \frac{\partial z}{\partial x} = 2xy^2 = 2x^5$$

Relationships:

$$\begin{tikzcd}
 & z \arrow[dl, dash] \arrow[dr, dash] & \\
x \arrow[d, dash, "x=x"'] & & y \arrow[d, dash, "y=x^2"] \\
x & & x
\end{tikzcd}$$